that the small signal gain is preserved. It may also be accomplished by choosing a diode with a large Q_m/S_m ratio. In terms of diode parameters this ratio may be expressed as $2C_{min}^2 V_B$ [2]. Therefore, the diode should have a large breakdown voltage V_B and largest possible capacitance at V_B.

If the values for A_0 and A_1 (10) are substituted into (17), it becomes

$$P_{0m} = \frac{V_g^2}{2R_L} \frac{\omega_3}{\omega_2} = P_{P\,\text{avail.}} \frac{\omega_3}{\omega_2} \cdot \tag{38}$$

It has been assumed that the sources have been matched to the diode impedances (i.e., $R_{T_3}=2R_L$, $R_{T_2}=2R_{g_2}$). The maximum obtainable output power is therefore the available pump power multiplied by the ratio of output frequency to pump frequency. If the pump power is increased 10 dB, P_{0m} also increases 10 dB. By examining (36), it is seen that increasing P_{0m} 10 dB decreases the first intermodulation distortion product by 20 dB. The intermodulation distortion of a parametric upconverter can therefore be reduced by increasing the pump power; for every one-decibel increase in pump power the first intermodulation distortion product is reduced two decibels.

Intermodulation distortion can be predicted simply by measuring the transfer characteristic, i.e., P_0 vs. P_{in}, and the frequency response of the device under test and then using (36). To compensate for the frequency response, (36) becomes

$$IMR_1(\text{dB}) = 2\frac{P_0}{P_{0m}}(\text{dB}) - 19.5 - B, \tag{39}$$

where B is the attenuation of the bandpass characteristic at the intermodulation frequencies.

Equation (37) should not be considered the result for this particular device only. The intermodulation analysis was done for the gain equation, and is applicable to any device which has a gain equation of the same form.

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The Effect of Parasitic Elements on Reflection Type Tunnel Diode Amplifier Performance

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Abstract—The effect of the tunnel diode series inductance and stray capacitance on the gain and bandwidth of broadband reflection type amplifiers is considered. General stability criteria imposed by these reactances are given together with realizability conditions for ideal (flat gain), Butterworth and Chebyshev responses. The main effect of the parasitic elements is to restrict the range of gain and bandwidth which may be achieved for a given number of elements in the matching network. The minimum gain is restricted together with both the maximum and minimum bandwidths. Comprehensive sets of curves are given which enable a rapid design of either Butterworth or Chebyshev response to be accomplished, and a procedure is given for conversion of the low-pass prototype network to band-pass form in the presence of the parasitic reactances. The frequency transformation is used to obtain an upper limit on the center frequency of the band-pass amplifier imposed by the parasitics. The use of the design data is illustrated by numerical examples.

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I. Introduction

The BROADBAND tunnel diode reflection type amplifier has now reached the stage where it has become a competitor for many applications in microwave systems. With the production of diodes having ever increasing cutoff frequencies operation at X band and above is presently possible. In such high frequency diodes the junction capacitance is usually rather small being typically less than 0.5 pF, with the result that the parasitic elements of the diode (i.e., the series inductance and package capacitance) have reactances at the operating frequency which are of the same order as the reactance of the junction capacitance.

The synthesis of tunnel diode amplifiers has hitherto mainly depended on representing the diode as the parallel combination of a frequency independent capacitance and negative resistance over the frequency band of interest [1]–[5], and from this representation a lossless coupling network, to be inserted between the diode and the circulator, is synthesized. However, if the diode series inductance and stray (package) capacitance have significant reactances at the operating frequency, the representation of the diode as a frequency independent capacitance and negative resistance will no longer be valid over a wide bandwidth. Furthermore the parasitic reactances introduce additional limitations on gain, bandwidth, and stability which, of course, are not revealed when the simplified diode model is used. Smilen [6] has derived some limitations imposed by the series inductance alone, where he shows from Fano's results [7] that if this inductance is above a particular value, instability inevitably results. He also points out that even when the inductance is less than this value, it limits the absolute maximum bandwidth which can be realized. A design procedure for use with large negative conductance diodes has recently been given [8].

In this paper more detailed consideration is given to the effect of the series inductance alone, and the limitations imposed by the combination of the series inductance and stray capacitance are fully investigated. The problem of transforming the low-pass prototype networks, resulting from the synthesis procedures, to band-pass form in the presence of the parasitic reactances is also considered. Comprehensive design curves are given which enable a rapid design to be accomplished for a particular diode and amplifier specification.

II. The Low-Pass Prototype Coupling Network

A network representation of the reflection type tunnel diode amplifier is shown in Fig. 1. N' is a lossless coupling network, and N_D represents the reactive elements associated with the diode. The synthesis problem is to find a network N' which, in cascade with N_D, realizes an overall network N such that the magnitude of the reflection coefficient $|\rho|$ at port 2 of the circulator, which is the gain of the amplifier, is a prescribed function of frequency. As usual it is convenient to synthesize N' as a low-pass network and afterwards perform a low-pass to band-pass transformation in order to realize the required band-pass amplifier. Ideally in the low-pass prototype the reflection coefficient should have a prescribed constant value (greater that unity) up to the cutoff frequency and be unity beyond this. In practice this performance cannot be achieved with a finite matching network so the ideal behavior is approximated by suitable functions which are commonly of the Butterworth or Chebyshev type.

In order to apply the techniques of passive network synthesis to the problem it is convenient to define a reflection coefficient ρ' corresponding to the network N being terminated in a positive resistance R in place of $-R$. It is easy to show [2] that

$$|\rho'| = \frac{1}{|\rho|}. \tag{1}$$

The ideal performance for $|\rho|$, then corresponds to a function $|\rho'|$ which is constant and less than unity in the pass band, and unity outside the pass band. The Chebyshev approximation for $|\rho'|$ is, following the notation of [5],

$$|\rho'|^2 = \frac{k^2 + h^2 Tn^2(\omega)}{1 + k^2 + h^2 Tn^2(\omega)}. \tag{2}$$

For the Butterworth response it is convenient to write the functions in the following manner:

$$|\rho'|^2 = \frac{K^{2n} + \alpha^{2n}\omega^{2n}}{1 + \alpha^{2n}\omega^{2n}} \tag{3}$$

where

$$K \leq 1$$

so that

$$|\rho|^2 = \frac{1 + \alpha^{2n}\omega^{2n}}{K^{2n} + \alpha^{2n}\omega^{2n}} = G.$$

K determines the maximum gain (at $\omega = 0$) and α determines the ratio of the gain at $\omega = 1$ to the maximum gain. In fact

$$K = G_{\max}{}^{-1/2n} \tag{4}$$

and

$$\alpha = \left(\frac{\delta - 1}{G_{\max} - \delta}\right)^{1/2n} \tag{5}$$

where δ is the ratio of the maximum gain to the gain at $\omega = 1$. This enables Chebyshev and Butterworth responses to be compared since δ in the Butterworth functions as defined above, corresponds to the ripple in the Chebyshev case.

III. Integral Restrictions on Gain and Bandwidth

Integral restrictions derived by Fano [7] apply to the passive reflection coefficient ρ', and by means of (1) these can be extended [5] to the tunnel diode reflection coefficient ρ. However in order to do so one approximation is required, namely that the portion of the diode shown in Fig. 2(a) must be represented as a parallel combination of an equivalent negative resistance and capacitance $-R$, C_d as shown in Fig. 2(b). R and C_d are frequency dependent, but provided the operating band of frequencies is well below the resistive cutoff frequency these parameters are relatively frequency insensitive. This is, however, not true if the diode inductance is included in an equivalent parallel RC combination. The network N_D can then be represented as shown in Fig. 2(c).

Because of the three reactive elements in N_D three integral restrictions are placed on $|\rho|$. These are [7] (using Fano's notation),

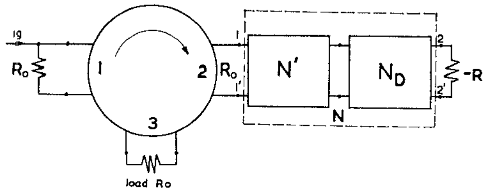


Fig. 1. Circulator and tunnel diode with coupling network N'.

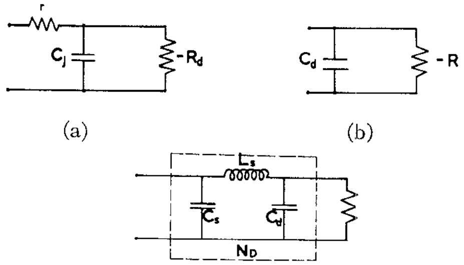


Fig. 2. (a) Equivalent circuit of tunnel diode junction. (b) Parallel RC representation of junction. (c) Equivalent circuit including parasitic elements.

$$\int_0^\infty \ln |\rho| \, d\omega' = \frac{\pi}{RC_d}$$

$$\int_0^\infty \omega'^2 \ln |\rho| \, d\omega' = \frac{\pi}{3}\left(\frac{3R^2C_d - L_s}{L_sR^3C_d^3}\right)$$

$$\int_0^\infty \omega'^4 \ln |\rho| \, d\omega' = \frac{\pi}{5}\left(\frac{L_s^2C_s - 5L_sC_sC_dR^2 + 5(C_s + C_d)C_d^2R^4}{L_s^2C_sC_d^5R^5}\right). \quad (6)$$

The equality signs appear above because right half plane zeros of ρ' are not permitted since they correspond to right half plane poles of ρ which would indicate instability. Equations (6) may be conveniently rewritten using the following normalization:

$$L_s \to L = \frac{L_s}{R^2C_d}$$

$$C_s \to C = \frac{C_s}{C_d}$$

$$\omega' \to \omega = \omega' RC_d$$

so that

$$R \to 1$$

$$C_d \to 1$$

and N_D of Fig. 2(c) assumes the form shown in Fig. 3. Equations (6) then become

$$\int_0^\infty \ln |\rho| \, d\omega = \pi \quad (7a)$$

$$\int_0^\infty \omega^2 \ln |\rho| \, d\omega = \frac{\pi}{3}\left(\frac{3 - L}{L}\right) \quad (7b)$$

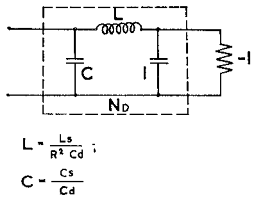


Fig. 3. Normalized equivalent circuit of tunnel diode.

$$\int_0^\infty \omega^4 \ln |\rho| \, d\omega = \frac{\pi}{5}\left(\frac{L^2C - 5LC + 5C + 5}{L^2C}\right). \quad (7c)$$

All three equations must be satisfied simultaneously. This, in general, means that the behavior of $|\rho|$ as a function of frequency must be such as to satisfy these equations. Since L is an inaccessible parameter and C may be increased but not decreased, the diode parameters limit the types of response which may be realized.

A. Stability

It is obvious that in each of (7) the right-hand side must be positive if a stable amplifier is to be at all possible since no specific restriction is made at this stage on the gain-frequency characteristics. This requirement gives

$$L < 3 \quad (8a)$$

and

$$C(L^2 - 5L + 5) > -5 \quad (8b)$$

from (7b) and (7c). Equation (8a) means that if $L_s > 3R^2C_d$ it is impossible to construct a stable amplifier, as pointed out by Smilen [6]. Even if equation (8a) is satisfied instability may yet result from violation of (8b). This equation is best interpreted graphically. The function

$$\frac{-5}{L^2 - 5L + 5}$$

is plotted in Fig. 4. For $L < \frac{1}{2}(5 - \sqrt{5})$, (8b) is satisfied by any positive value of C. If $L > \frac{1}{2}(5 - \sqrt{5})$ then C is restricted to lie below the curve shown. L must of course be less than 3 to satisfy (8a). If $C < 4$ then (8b) will be satisfied irrespective of the value of L. It should be noted that (8a) and (8b) are necessary for stability, but may not in fact be sufficient in particular cases. This question will be considered in later sections.

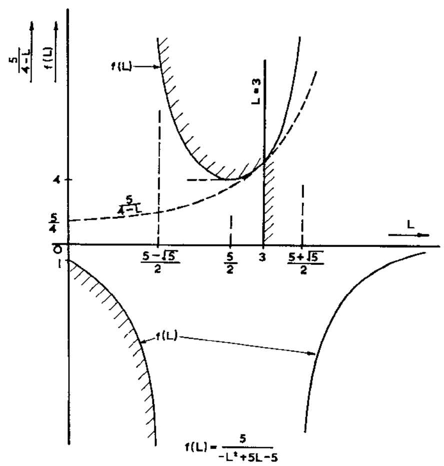


Fig. 4. Stability and realizability limits for tunnel diode amplifier.

B. Idealized Response

In order that the idealized response previously mentioned ($|\rho|$ constant and greater than unity in the pass-band and unity outside it) may be realized, the network N' of Fig. 1 must contain an infinite number of elements. In this case (7) becomes (for a low-pass prototype response)

$$\omega_c \ln |\rho| = \pi \tag{9a}$$

$$\omega_c^3 \ln |\rho| = \pi\left(\frac{3-L}{L}\right) \tag{9b}$$

$$\omega_c^5 \ln |\rho| = \pi\left(\frac{L^2C - 5LC + 5C + 5}{L^2C}\right). \tag{9c}$$

The value of C required in order that all three equations be simultaneously satisfied is given by

$$C = \frac{5}{4-L}$$

and is plotted in Fig. 4. This is the condition that the ideal response be realizable. If the value of C provided by the diode alone is less than the value previously given then it may be increased to the required level, but if it is greater than this value the ideal response is not possible. Since the curve for $C=5/(4-L)$ in Fig. 4 lies below the stability curve (for $C>0$), the value of C satisfies the stability requirements.

In an idealized band-pass response the restrictions become

$$(\omega_2 - \omega_1) \ln |\rho| = \pi \tag{10a}$$

$$(\omega_2^3 - \omega_1^3) \ln |\rho| = \pi\left(\frac{3-L}{L}\right) \tag{10b}$$

$$(\omega_2^5 - \omega_1^5) \ln |\rho| = \pi\left(\frac{L^2C - 5LC + 5C + 5}{L^2C}\right) \tag{10c}$$

where ω_2 and ω_1 are the upper and lower edges of the pass band, respectively. If we then put

$$\omega_c = \omega_2 - \omega_1$$

and

$$\omega_0^2 = \omega_1\omega_2$$

(10), after some manipulation, can be reduced to the same form as (9) if L is replaced by

$$\frac{L}{1 - \omega_0^2 L}$$

and C by

$$\frac{C(1-\omega_0^2 L)^2}{1 - LC\omega_0^2 + L^2C\omega_0^4}.$$

Thus the higher the center frequency of the band-pass response, the smaller the bandwidth for a given gain. In fact the stability restrictions of (8) may be rewritten as upper bounds on the operating frequency by substituting the values for L and C previously given. This yields

$$\omega_0^2 < \frac{3-L}{3L} \tag{11a}$$

and

$$\frac{L^2C - 5LC + 5C + 5}{L^2C} + 10\omega_0^4 + 5\omega_0^2 - \frac{15\omega_0^2}{L} > 0. \tag{11b}$$

The value of C required to ensure that all three restrictions given in (10) are simultaneously satisfied can be found as

$$C = \frac{5}{(1 - L\omega_0^2)(4 - L + L\omega_0^2)}. \tag{12}$$

Substitution of this value into (11b) shows that the expression given is always a perfect square and therefore this stability restriction is invariably satisfied. It can also be shown that the value of C given in (12) is posi-

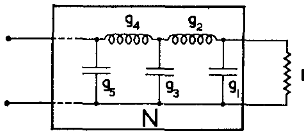


Fig. 5. Low-pass prototype circuit for tunnel diode amplifier.

tive provided that $\omega_0^2 < 1/L$, which is less severe than (11a). Thus the upper limit on the center frequency of a band-pass amplifier with ideal response is that given by (11a), which, of course, requires also that $L<3$ in accordance with the necessary stability requirement.

As an example of the application of the limits derived in this section consider a diode with $RC_d = 2\times 10^{-11}$, $L=0.4$, $C=1.2$, $f_0'=10$ Gc/s, which might be typical of an X-band model. The stability requirements of (8) turn out to be

$$L = 0.4 < 3$$

and

$$C(L^2 - 5L + 5) = 3.8 > -5$$

which are obviously satisfied.

The normalized ω_0 is 1.26 which is less than $(3-L/3L)^{1/2}$ so that an ideal response can be realized. Finally the value of C required by (12) is 3.22 so that the diode falls within this realizability restriction.

IV. Butterworth Response

Restrictions similar to those obtained for the idealized response (requiring an infinite number of circuit elements) may also be derived for any amplifier which is required to realize a Butterworth gain response of a particular order. The treatment is given in terms of the low-pass prototype amplifier both for the Butterworth response considered in this section and the Chebyshev response which is discussed in the next section. The use of these results for band-pass amplifiers will become clear in Section VI which considers the low-pass to band-pass transformation.

The most convenient starting point for discussion of Butterworth amplifiers is the set of recurrence relationships for the element values of the low-pass prototype network. These are [9]

$$g_1 = \frac{2\alpha \sin \frac{\pi}{2n}}{1-K} \tag{13a}$$

$$g_r g_{r+1} = \frac{4\alpha^2 \sin (2r-1)\frac{\pi}{2n} \sin (2r+1)\frac{\pi}{2n}}{1 - 2K\cos\frac{r\pi}{n} + K^2}$$

$$r = 1, 2, \cdots, n-1 \tag{13b}$$

where the response is that given in (3) and the g_r refer to the element values (normalized L or C) of the prototype network shown in Fig. 5. In the case of the tunnel diode the reactive elements determine g_1, g_2, and g_3 as follows: if ω_c is the normalized bandwidth (for some particular attenuation compared to the dc gain) we have

$$g_1 = \omega_c$$
$$g_2 = L\omega_c$$
$$g_3 = C\omega_c$$

so that (13a) and (13b) become

$$\omega_c = \frac{2\alpha \sin\frac{\pi}{2n}}{1-K} \tag{14a}$$

$$L\omega_c^2 = \frac{4\alpha^2 \sin\frac{\pi}{2n}\sin\frac{3\pi}{2n}}{1-2K\cos\frac{\pi}{n}+K^2} \tag{14b}$$

$$\omega_c^2 LC = \frac{4\alpha^2 \sin\frac{3\pi}{2n}\sin\frac{5\pi}{2n}}{1-2K\cos\frac{2\pi}{n}+K^2} \tag{14c}$$

as before these must be simultaneously satisfied.

Using (14a) and (14b) and applying the condition that $0 \le K \le 1$ (i.e. ω_c positive, gain positive) one finds that a condition for realizability of Butterworth response is

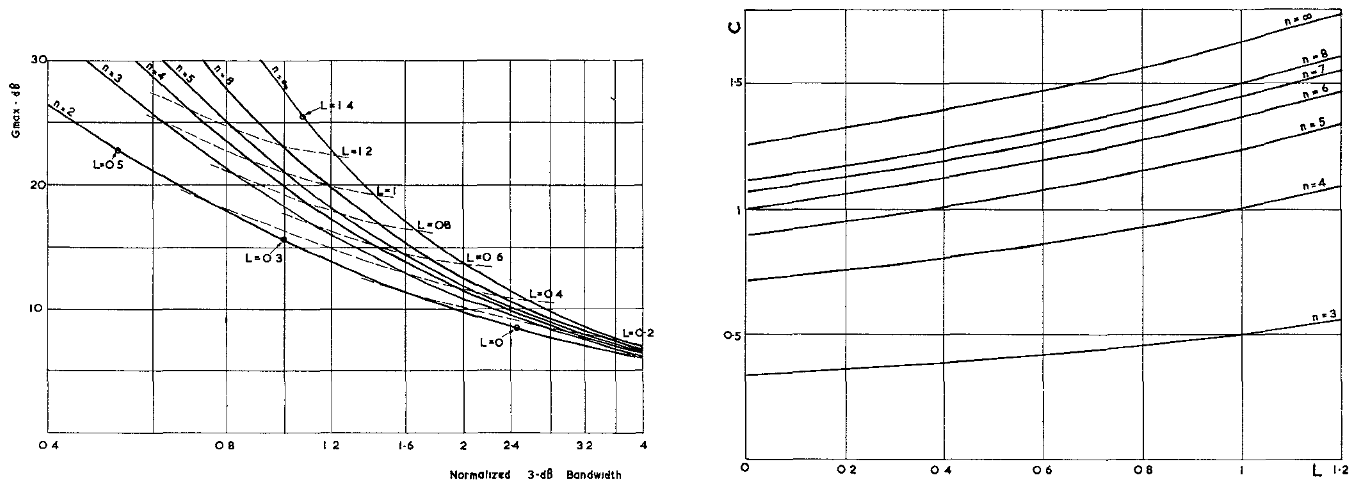
$$L \le 4\cos^2\frac{\pi}{2n} - 1 \tag{15}$$

which as $n \to \infty$ to give the idealized response of the previous section, becomes $L \le 3$ as before.

In order that all of (14) be simultaneously satisfied a relationship between L and C must exist. This can be found by eliminating K and ω_c/α from (14) to be

$$C = \frac{\sin\frac{5\pi}{2n}}{\sin\frac{\pi}{2n}\left(4\cos^2\frac{\pi}{2n} - L\right)} \tag{16}$$

which in the idealized case $(n\to\infty)$ becomes $C=5/(4-L)$. If the value provided by the diode alone is less than this, it is possible to construct a Butterworth amplifier, while if it is greater, then this form of amplifier is impossible. The smaller the value of n $(n\ge 3)$ the more severe the restrictions on both L and C so that in practice these limitations would usually mean that a Butterworth amplifier could be constructed only if at least a certain number of elements are used. With a given diode which meets the restrictions discussed above



(a) (b)

Fig. 6. Design curves for amplifiers with Butterworth response. (a) Gain-bandwidth. (b) Capacitance-inductance.

the values of K and ω_c/α can be found from (14), and once n is chosen the remaining elements of the matching network N' of Fig. 1 can be found by repeated application of (13b).

Usually in Butterworth amplifiers the 3-dB bandwidth is the one of interest. In this case $\delta = 2$, and substitution of this value in (5) gives

$$\alpha = \frac{K}{(1 - 2K^{2n})^{1/2n}}\,. \tag{17}$$

From (17) and (14a) and substituting for K from (4) one finds that

$$\omega_c[G_{\max}^{1/2n} - 1]\left[1 - \frac{2}{G}\right]^{1/2n} = 2 \sin \frac{\pi}{2n} \tag{18}$$

which is a generalization of the simple expression derived in [5], Appendix I. Of course it is required in addition to (18) that the constraints on L and C be fulfilled. This equation is equivalent to a result derived by Youla and Smilen [4].

Figure 6(a) shows curves of gain as a function of the 3-dB bandwidth given by (18), with contours of constant L superimposed. Figure 6(b) shows the relationship between L and C given by (16). For a given diode, L is inaccessible and so operation is confined to the particular constant L contour. The minimum allowable value of n is determined from Fig. 6(b). As can be seen from Fig. 6(a) the effect of the parasitics is to limit the range of bandwidths which can be achieved by varying the number of elements in the matching network. As an application of the design information given in Fig. 6 consider the same diode as used in the example given in the previous section. It can be seen that the minimum permissible n for $C = 1.2$ and $L = 0.4$ is 8. This gives a gain of 10.95 dB and a 3-dB bandwidth of 2.3 for the low-pass prototype.

V. Chebyshev Response

In this section the particular limitations imposed by the requirement of a Chebyshev response are considered. As before the most convenient starting point is the set of recurrence relationships [9] for the element values of the low-pass prototype network. These are

$$g_1 = \frac{2 \sin \dfrac{\pi}{2n}}{x - y}$$

$$g_r g_{r+1} = \frac{4 \sin \dfrac{2r-1}{2n}\pi \sin \dfrac{2r+1}{2n}\pi}{x^2 + y^2 + \sin^2 \dfrac{r\pi}{n} - 2xy \cos \dfrac{r\pi}{n}}$$

$$r = 1, 2, 3 \cdots n - 1. \tag{19}$$

If

$$\frac{1 + k^2}{h^2} = \sinh^2 na$$

and

$$\frac{k^2}{h^2} = \sinh^2 nb$$

x and y are given by

$$x = \sinh a = \sinh \frac{1}{n} \sinh^{-1} \left\{\frac{G_{\max} - \delta}{\delta - 1}\right\}^{1/2}$$

$$y = \sinh b = \sinh \frac{1}{n} \sinh^{-1} \left\{\frac{G_{\max} - \delta}{G_{\max}(\delta - 1)}\right\}^{1/2} \tag{20}$$

and the gain response is that given in (2). $G_{\max}$ is the maximum pass-band gain, and δ is the ripple [5], where the notation is that used by Fano [7]. In the case of the tunnel diode we have

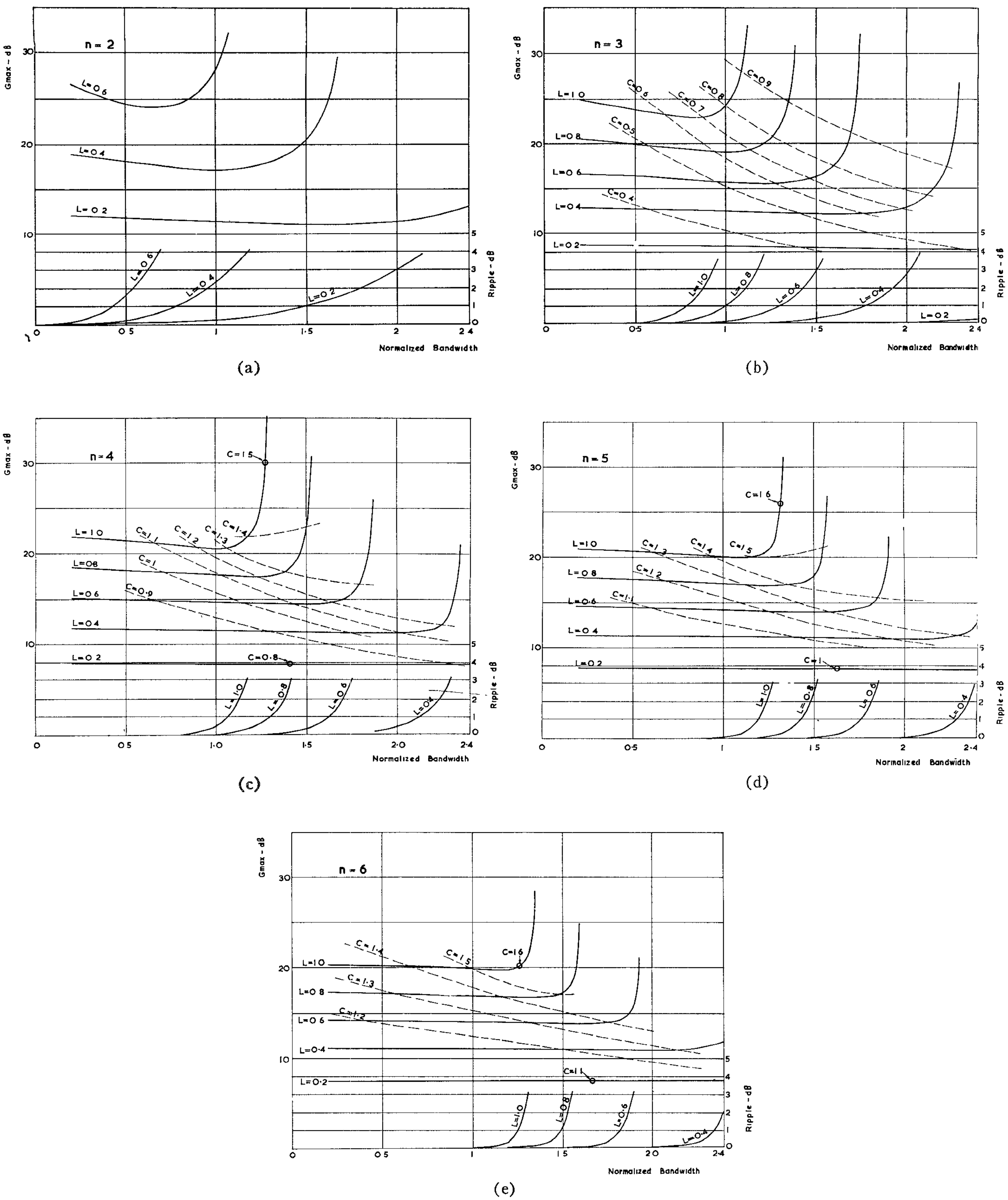


Fig. 7. Design curves for amplifiers with Chebyshev response. (a) $n=2$. (b) $n=3$. (c) $n=4$. (d) $n=5$. (e) $n=6$.

$$\omega_c = \frac{2 \sin \frac{\pi}{2n}}{x - y} \tag{21a}$$

$$L\omega_c^2 = \frac{4 \sin \frac{\pi}{2n} \sin \frac{3\pi}{2n}}{x^2 + y^2 + \sin^2 \frac{\pi}{n} - 2xy \cos \frac{\pi}{n}} \tag{21b}$$

$$\omega_c^2 LC = \frac{4 \sin \frac{3\pi}{2n} \sin \frac{5\pi}{2n}}{x^2 + y^2 + \sin^2 \frac{2\pi}{n} - 2xy \cos \frac{2\pi}{n}} \cdot \tag{21c}$$

These three equations must be simultaneously satisfied, so that as before there will be an upper limit on L and a relationship between C and L.

From (20) and (21a) it is seen that for Chebyshev response the following inequality must hold:

$$x \geq y \geq 0$$

and when this condition is applied to (21b) we obtain

$$L \leq \frac{4 \cos^2 \frac{\pi}{2n} - 1}{1 + \omega_c^2 \cos^2 \frac{\pi}{2n}} \cdot \tag{22}$$

Solving (21) simultaneously one obtains the relationship between C and L as

$$C = \frac{\sin \frac{5\pi}{2n}}{\sin \frac{\pi}{2n} \left\{ 4 \cos^2 \frac{\pi}{2n} - L\left(1 + \omega_c^2 \sin^2 \frac{\pi}{n}\right)\right\}} \cdot \tag{23}$$

C is a function of ω_c in addition to L because of the additional parameter (the ripple) as compared to the Butterworth response. If $\omega_c \rightarrow 0$, implying zero ripple, C assumes the value required for a Butterworth response, as is also the case for L. Figure 7(a) through (e) show curves of maximum gain and ripple against bandwidth for $n = 2$ through 6 and various values of L with contours of constant C (except for $n = 2$) superimposed. The use of these curves will now be illustrated by an example. We take the same diode as before and let us take as a specification $G_{max} \geq 10$ dB, $\omega_c = 2.3$, ripple ≤ 1.5 dB. It can be seen from Fig. 7(d) that $n = 5$ meets this specification requiring $C = 1.3$, and giving a ripple of 1.2 dB with a maximum gain of 11.5 dB.

VI. Low-Pass to Band-Pass Transformation with Series Inductance and Stray Capacitance

In order that a band-pass amplifier be constructed with the same bandwidth as the low-pass prototype it is necessary to tune each L and C in the total network N of Fig. 1 to the desired center frequency. In the case of the tunnel diode, however, the junction capacitance and series inductance are inaccessible and must remain untuned in the transformation. It is therefore necessary to find a network which when terminated in the tunnel diode realizes the bandwidth conserving transformation. Figure 8 shows the network which would result if it were possible to tune all elements in the equivalent circuit so that we must find a network of the type shown in Fig. 9 which is exactly equivalent to Fig. 8. It is shown in the appendix that the portion of the circuit in Fig. 8 within the dotted lines is equivalent to the corresponding portion of that in Fig. 9 if

$$L'' = \frac{L'}{1 + L'\omega_0^2} \tag{24}$$

and

$$C'' = C' \frac{(1 + L'\omega_0^2)^2}{1 + L'C'\omega_0^2} \cdot \tag{25}$$

But since the circuit of Fig. 9 must have the actual tunnel diode as its termination $C'' = C$ and $L'' = L$ where C and L are the tunnel diode parameters. Inversion of (24) and (25) gives

$$L' = \frac{L}{1 - L\omega_0^2} \tag{26}$$

$$C' = \frac{C(1 - L\omega_0^2)^2}{1 - LC\omega_0^2 + L^2C\omega_0^4} \tag{27}$$

which are the same as the values obtained in the particular case of the ideal response previously considered. Now the circuit of Fig. 8 has the same bandwidth as the low-pass prototype amplifier terminated in a tunnel diode whose parameters are L' and C' so that in order to calculate the bandwidth, gain, etc. of a band-pass amplifier using the actual tunnel diode with parameters L and C we need only calculate the corresponding quantities for the low-pass prototype with parameters L' and C' as given by (26) and (27). The effect of this transformation is to produce poorer performance as the center frequency of the band-pass amplifier is increased. By combining (26) and (27) with (22) and (23) corresponding restrictions to these can be found for realizability of

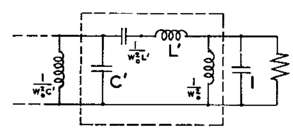


Fig. 8. Ideal circuit for band-pass tunnel diode amplifier.

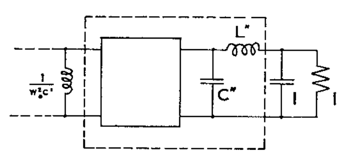


Fig. 9. Realizable form of circuit for band-pass amplifier.

band-pass amplifiers, the Butterworth case being obtained by setting $\omega_c = 0$, these restrictions can be expressed as an upper bound on ω_0 to give

$$\omega_0^2 \le \frac{4\cos^2\dfrac{\pi}{2n} - 1 - L\left(1 + \omega_c^2\cos^2\dfrac{\pi}{2n}\right)}{L\left(4\cos^2\dfrac{\pi}{2n} - 1\right)}. \quad (28)$$

For the diode considered in the previous examples operating at 10 Gc/s we have $\omega_0 = 1.26$, which gives $L' = 1.1$ and $C' = 0.23$. These values would then be used in conjunction with Figs. 6 and 7 to determine the performance of the band-pass amplifier.

Having used the various procedures outlined so far one arrives at a lumped element band-pass amplifier meeting the design specifications. The remaining steps consist in determining approximations to these lumped resonators and transformer in distributed form by standard methods, and in determining the required circulator impedance using the methods previously given [5].

VII. Conclusions

A design theory has been given for broadband reflection type tunnel diode amplifiers which takes account of the parasitic elements of the diode. These elements are often the limiting factors for operation at X band and above. Curves have been presented which enable a rapid design of Butterworth or Chebyshev gain responses to be achieved. The main effect of the parasitic elements is to determine both gain and bandwidth for a given number of elements, in the Butterworth case, and to limit the variation of these quantities (and the ripple) in the Chebyshev case. Stability limitations are also imposed, together with limitations which may prohibit the realization of one or both of the responses mentioned. These limitations also have significance from the diode designer's point of view, and are fully described in the text. A transformation from the low-pass prototype network to a band-pass equivalent is described, and used to find an upper limit on the center frequency of the band-pass amplifier, imposed by the parasitics.

Appendix

In order to synthesize a circuit equivalent to the portion of Fig. 8 enclosed within the dashed lines we formulate the

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

or chain matrix for this section, which is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ C'S & 1 \end{bmatrix} \begin{bmatrix} 1 & L'S + \dfrac{L'\omega_0^2}{S} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{\omega_0^2}{S} & 1 \end{bmatrix}$$

$$= \begin{pmatrix} 1 + L'\omega_0^2 + \dfrac{L'\omega_0^4}{S^2} & L'S + \dfrac{L'\omega_0^2}{S} \\ C'S(1 + L'\omega_0^2) + \dfrac{\omega_0^2(1 + L'C'\omega_0^2}{S} & 1 + L'C'S^2 + L'C'\omega_0^2 \end{pmatrix}.$$

Now the portion of the network in Fig. 9 within the dotted lines must produce the same number of zeros of transmission at zero and infinite frequencies as the corresponding portion of Fig. 8. That is, there must be two zeros of transmission at the origin and two at infinity. L'' and C'' produce the pair at infinity so that the total circuit must take the form shown in Fig. 10. The elements C_1 and L_1 produce the transmission zeros at the origin and the transformer is introduced for generality.

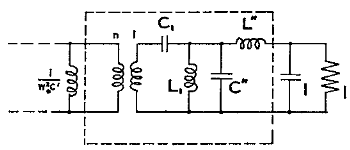


Fig. 10. Detailed form of circuit for band-pass amplifier.

The inductance L_1 cannot be placed on the transformer side of C_1 since then it would merely add to the inductance $1/C'\omega_0^2$ and fail to produce the required zero of transmission. The chain matrix of the portion of Fig. 10 within the dotted lines is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} n\left(1 + \dfrac{C''}{C_1}\right) + \dfrac{n}{L_1C_1S^2} & nL''S\left(1 + \dfrac{C''}{C_1}\right) + \dfrac{n}{C_1S}\left(1 + \dfrac{L''}{L_1}\right) \\ \dfrac{C''}{n}S + \dfrac{1}{nL_1S} & \dfrac{1}{n}\left(1 + \dfrac{L''}{L_1}\right) + \dfrac{L''C''}{n}S^2 \end{pmatrix}$$

and equating like coefficients in the two matrices one obtains

$$n = \frac{1 + L'\omega_0^2}{1 + L'C'\omega_0^2}$$

$$C_1 = \frac{(1 + L'\omega_0^2)^2}{L'\omega_0^2(1 + L'C'\omega_0^2)}$$

$$L_1 = \frac{1}{\omega_0^2(1 + L'\omega_0^2)}$$

$$L'' = \frac{1}{1 + L'\omega_0^2}$$

$$C'' = C' \frac{(1 + L'\omega_0^2)^2}{1 + L'C'\omega_0^2}.$$

The latter two equations being those used in the text.

In particular cases it may be found possible to absorb the transformer at some position in the remainder of the network, but it is difficult to formulate meaningful general rules.

Since submission of this paper it has been drawn to the authors' attention that the low-pass to band-pass transformation where the diode series inductance is considered has been given by Dr. L. I. Smilen in the Polytechnic Institute of Brooklyn, N. Y., Progress Report No. 26, April 1, 1964 through September 30, 1964.

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Locking of Magnetrons by an Injected RF Signal

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***Abstract*—An equivalent circuit is given which quantitatively predicts the performance of magnetron oscillators when they are frequency locked by an injected RF signal. A method is presented for the reciprocal coupling of magnetrons to a traveling wave without reflection. The theory is supported by experimental results which include:**

1) a single-tube locked oscillator with nonreciprocal (circulator) coupling,
2) a three-tube locked oscillator array with reciprocal coupling,
3) a two-tube oscillator with reciprocal coupling.

The feasibility of various locked oscillator and self-oscillating arrays, including the effect of mismatched loads, is discussed.

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INTRODUCTION

THE ABILITY to lock the frequency of magnetron oscillators allows the application of these high-efficiency devices to systems such as pulse compression or frequency diversity radars which require electronic frequency control or radars which require pulse-to-pulse phase coherence. Furthermore, this ability allows the formation of an array of magnetrons having a coherent output power greater than a single tube. Such an array may consist of separate tubes connected by external circuitry or an extended interaction space within a single vacuum envelope.

The general concept of injection locking an oscillator has been studied by a number of investigators. Adler [1], for example, has made a general analysis of the